TWISTING OF A CIRCULAR RING
CALCULATION OF LOCAL STRESSES

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Project: IMPROVEMENTS OF FUTURE DEVELOPMENTS

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ABSTRACT:

This method of calculating local stresses in plates welded to the core and jacket of a jacketed pipng system is based on Timoshenko's: 'Strength of materials'.

This report is the result of a technical discussion between Tom Muilman of Stoomwezen BV and Paul W.H. Voorhaar.
This is to show the way of working only. The end results are intentionally left away to avoid that this calculation method will be used by third party personel.

My intention has been to proof that "Design by Analysis" needs a clear understanding of the calculation method and the boundary conditions.
If also a "Design by Rule" is required by the Client than intensive communication between the Notified Body and the engineer is a "must" in those cases where the Code does not (fully) covers the problem under consideration.

All positive contributions are welcome.

REFERENCES:

T. Muilman Mui970210 rev.A 10 maart 1997
S. Timoshenko Strength of materials part II 3rd edition page 140-143 par.28
S. Timoshenko Theory of Plates and Shells 2nd edition page 468-469

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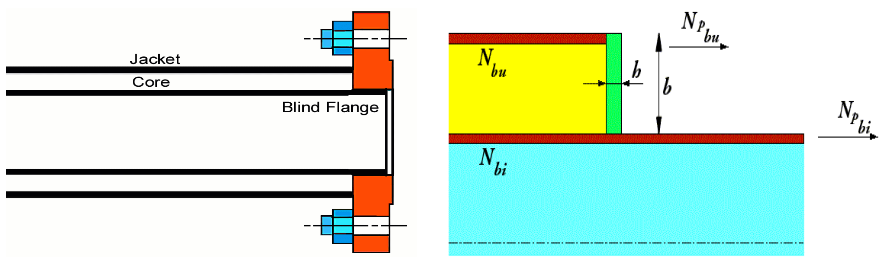
Calculation of a circular ring twisted by uniformly distributed couples

(Kantelschijf berekening)

Introduction

The purpose of this calculation is to work out the stresses in a jacketed piping system at the location of a flanged connection as shown in the next figure.



The system consists of a core and jacket pipe connected with a plate. If the jacket and core pipe are of the same material and have the same loading then no differential displacement will occur at the end-points of that pipe. Connecting the two pipes by means of a plate will create no stresses. However if the materials of the outer and inner pipe are different, or if there is a difference in loading, the outer-pipe will want to displace different from the inner-pipe. Here the connection between the two pipes will create local stresses.

The method used is based on:
(Lit.1) S. Timoshenko 'Strength of materials part II' 3rd edition page 140-143 par. 28 and
(Lit. 2) S. Timoshenko 'Theory of Plates and Shells' 2nd edition page 468-469.

Depending on the boundary conditions we can use this method for:
1. Flanged connection. (*Core and Jacket pipe discontinue.*)
2. End plate. (*Jacket pipe discontinue, core pipe continues.*)
3. Connect plate. (*Core and Jacket pipe continues.*)

The connecting plate is attached to both pipes. Relative displacement will create stresses. If these stresses are too high they can only be reduced by decreasing the displacement difference.

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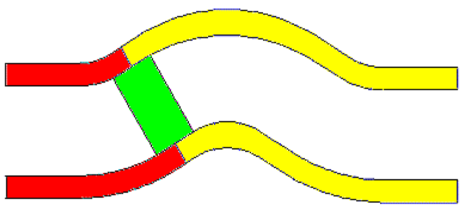
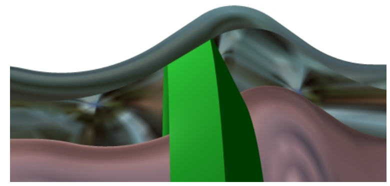
The relative displacement is highly influenced by the differential temperature of the two pipes. In the condition where the jacket pipe has a higher temperature than the core pipe one should consider that although the design temperatures can have a high difference, this is not always realistic. The product in jacket pipe will heat-up the jacket and core pipe since it is in direct contact with both pipes. The core pipe is in that condition cooled by the product stream. The jacket is cooled through the isolation layer.
If the stresses in this condition still are too high one can consider to electrically trace the jacket pipe during fabrication. In this way a 'cold spring' will be created and the pipe system will be pre-stressed.





Mathematical derivation of formulas
Based on the original document Mui970210 rev.A of Tom Muilman.

This theory is used to calculate the stresses in the inner pipe due to a plate welded in a jacketed system between inner and outer pipe.

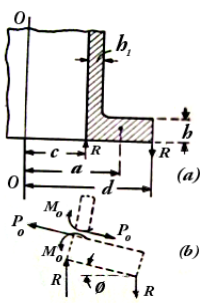


Fig. 95.

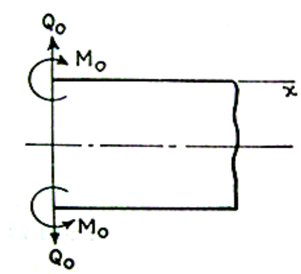


FIG. 236

List of symbols and definitions

Outside diameter	D	Plate thickness	t_{plate}	h
Outside diameter pipe	OD_{bi}	Wallthickness pipe	t_{bi}	h_1
Outside diameter jacket	OD_{bu}	Wallthickness jacket	t_{bu}	
Modulus of elasticity	E	External Pressure	p	
Modulus of elasticity pipe	E_{bi}	Pressure pipe	p_{bi}	
Modulus of elasticity jacket	E_{bu}	Pressure jacket	p_{bu}	
Modulus of elasticity plate	E_{plate}	Poisson ratio	ν	
Expansion	α	Radial displacement	ω	
Expansion	α_{bi}	Axial displacement	δ	Δ_{ax}
Expansion jacket	α_{bu}	Length of pipe or jacket	L	

The cylinder with free-end loaded with axis symetrical bending couple

We neglect the infuence of the longitudinal force for the poison effect only. Now we can determine th radial displacement of a cylinder with moment couple and shearforce according to lit. 2 page 469 - 475:

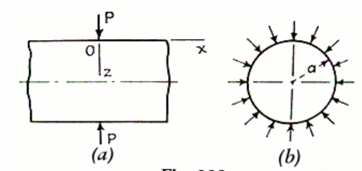


Fig. 238

The radius of the cylinder increases by the amount $\omega(p) := \frac{a^2 \cdot p}{E \cdot h}$

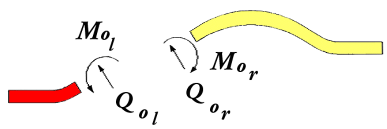
$$\beta := \left[\frac{3\cdot\left(1-\nu^2\right)}{a^2\cdot h^2}\right]^{0.25}$$ Lit 2 page 468 (275)

$$D := \frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}$$ Lit 2 page 468

$$\omega_{(x=0)} := -\frac{\beta\cdot M_o \cdot Q_o}{2\cdot\beta^3\cdot D} + \omega(p)$$ Lit 2 page 469 (279)

$$\left(\frac{dw}{dx}\right)_{x=0} := \frac{2\cdot\beta\cdot M_o + Q_o}{2\cdot\beta^2\cdot D}$$ Lit 2 page 470 (280)

When the load of a cylinder consists of a moment couple and a radial force at location 'S' we can determine the loadings in the two parts (**l**eft and **r**ight) according the following:



External loadings

The continuing cylinder with a loading consisting of axissymetrical bending

A continuing cylinder can be considered as two cylinders with a free-end conected to each other in location ‘S’ The rotations and radial displacements of both cylinders in ‘S’ are identical.
By doing so we have enough relations to solve the problem.

$M_o := M_{ol} - M_{or}$ or (I)

$M_{or} := M_{ol} - M_o$ (Ia)

$Q_o := Q_{ol} + Q_{or}$ or (II)

$Q_{or} := Q_o - Q_{ol}$ (IIa)

For the radial displacements formula (279) is used so:

$$\omega_l := -\frac{\beta \cdot M_{ol} \cdot Q_{ol}}{2 \cdot \beta^3 \cdot D} + \omega_l(p) \quad = \quad \omega_r := -\frac{\beta \cdot M_{or} \cdot Q_{or}}{2 \cdot \beta^3 \cdot D} + \omega_r(p) \qquad \text{(III)}$$

or

$$\beta \cdot M_{ol} + Q_{ol} - 2 \cdot \beta^3 \cdot D \cdot w_l(p) := \beta \cdot M_{or} + Q_{or} - 2 \cdot \beta^3 \cdot D \cdot w_r(p) \qquad \text{(IIIa)}$$

For the angular displacement we obtain:

$$\left(\frac{dw_l}{dx}\right) := \frac{2 \cdot \beta \cdot M_{ol} + Q_{ol}}{2 \cdot \beta^2 \cdot D} \quad = \quad -\left(\frac{dw_r}{dx}\right) := -\frac{2 \cdot \beta \cdot M_{or} + Q_{or}}{2 \cdot \beta^2 \cdot D} \qquad \text{(IV)}$$

or

$$2 \cdot \beta \cdot M_{ol} + Q_{ol} := -2 \cdot \beta \cdot M_{or} - Q_{or} \qquad \text{(IVa)}$$

When we substitude (Ia) and (IIa) in (IIIa) we find:

$$\beta \cdot M_{ol} + Q_{ol} := \beta \cdot \left(M_{ol} - M_o\right) + Q_o - Q_{ol} + 2 \cdot \beta^3 \cdot D \cdot \left(w_l(p) - w_r(p)\right)$$

or

$$2 \cdot Q_{ol} := -\beta \cdot M_o + Q_o + 2 \cdot \beta^3 \cdot D \cdot \left(w_l(p) - w_r(p)\right)$$

or

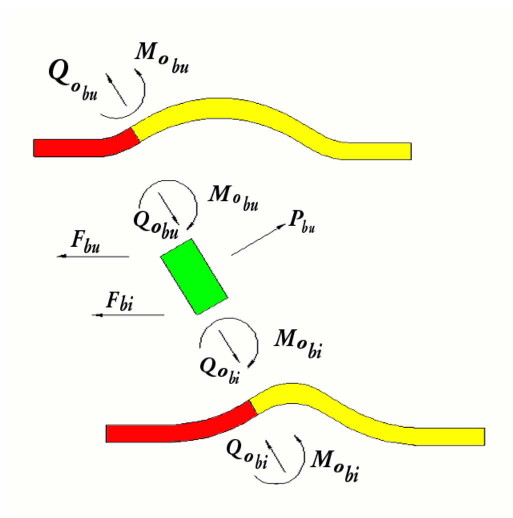
$$Q_{ol} := -\frac{1}{2} \cdot \beta \cdot M_o + \frac{1}{2} \cdot Q_o + \beta^3 \cdot D \cdot \left(w_l(p) - w_r(p)\right) \qquad \text{(V)}$$

When we substitude (Ia) and (IIa) in (IVa) we find:

$$2 \cdot \beta \cdot M_{ol} + Q_{ol} := -2 \cdot \beta \cdot \left(M_{ol} - M_o\right) - \left(Q_o - Q_{ol}\right)$$

or

$$4 \cdot \beta \cdot M_{ol} := 2 \cdot \beta \cdot M_o - Q_o \quad \text{or} \quad \beta \cdot M_{ol} := \frac{1}{2} \cdot \beta \cdot M_o - \frac{1}{4} \cdot Q_o \qquad \text{(VI)}$$



With equation (V) and (VI) we can wl from (II) express in M_o and Q_o:

$$w_l := w_r$$

$$w_r := -\frac{\beta \cdot M_{ol} + Q_{ol}}{2 \cdot \beta^3 \cdot D} + w_l(p)$$

$$w_r := -\frac{\frac{1}{2} \cdot \beta \cdot M_o - \frac{1}{4} \cdot Q_o - \frac{1}{2} \cdot \beta \cdot M_o + \frac{1}{2} \cdot Q_o + \beta^3 \cdot D \cdot \left(w_l(p) - w_r(p)\right)}{2 \cdot \beta^3 \cdot D} + w_l(p)$$

$$w_r := -\frac{\frac{1}{4} \cdot Q_o + \beta^3 \cdot D \cdot \left(w_l(p) - w_r(p)\right)}{2 \cdot \beta^3 \cdot D} + w_l(p)$$

$$w_r := -\frac{Q_o}{8 \cdot \beta^3 \cdot D} + \frac{1}{2} \cdot \left(w_r(p) - w_l(p)\right) + w_l(p)$$

$$w_r := -\frac{Q_o}{8 \cdot \beta^3 \cdot D} + \frac{1}{2} \cdot \left(w_r(p) + w_r(p)\right) \qquad \text{(VII)}$$

Likewise (V), (VI) and (IV) give:

$$\left(\frac{dw_l}{dx}\right) := -\left(\frac{dw_r}{dx}\right)$$

$$\left(\frac{dw_l}{dx}\right) := \frac{2 \cdot \beta \cdot M_{ol} + Q_{ol}}{2 \cdot \beta^2 \cdot D}$$

$$\left(\frac{dw_l}{dx}\right) := \frac{2 \cdot \left(\frac{1}{2} \cdot \beta \cdot M_o - \frac{1}{4} \cdot Q_o\right) - \frac{1}{2} \cdot \beta \cdot M_o + \frac{1}{2} \cdot Q_o + \beta^3 \cdot D \cdot \left(w_l(p) - w_r(p)\right)}{2 \cdot \beta^2 \cdot D}$$

$$\left(\frac{dw_l}{dx}\right) := \frac{\frac{1}{2} \cdot \beta \cdot M_o + \beta^3 \cdot D \cdot \left(W_l(p) - w_r(p)\right)}{2 \cdot \beta^2 \cdot D}$$

$$\left(\frac{dw_l}{dx}\right) := \frac{M_o}{4 \cdot \beta \cdot D} + \frac{1}{2} \cdot \beta \cdot \left(w_l(p) - w_r(p)\right)$$

We substitude:

$$\Delta\omega := w_l(p) - w_r(p) \qquad \text{(VIIIa)}$$

$$\left(\frac{dw_l}{dx}\right) := \frac{M_o}{4 \cdot \beta \cdot D} + \frac{1}{2} \cdot \beta \cdot \Delta\omega \qquad \text{(VIII)}$$

In Lit. 2 page 139 we find for the twisting ring:

$$\theta := \frac{12 \cdot M_t \cdot a_{twist}}{E \cdot h^3 \cdot \ln\left(\frac{d}{c}\right)}$$

With:

$$\gamma := E \cdot h^3 \cdot \ln\left(\frac{d}{c}\right) \qquad \text{(X)}$$

This becomes:

$$\theta := \frac{12 \cdot M_t \cdot a_{twist}}{\gamma} \qquad \text{(IX)}$$

M_t is the moment couple per unit length measured along the centercircle line as shown in figure 95. (Dimension a)

Connection of continues core pipe and jacket pipe with free end by means of a twisting ring

The loadings and their resulting angular displacement

The twisting ring is loaded by moment couples from the resulting forces F_{bu} and F_{bi} [N}, each with a moment arm ½b and the two resulting momemnt couple from the inner and outer tube M_{obu} and M_{obi}.

This results in a moment couple per unit length (acting on radius a):

$$M_t := \frac{(F_{bu} - F_{bi}) \cdot \frac{1}{2} \cdot b - M_{obu} \cdot 2 \cdot \pi \cdot R_{bu} - M_{obi} \cdot 2 \cdot \pi}{2 \cdot \pi \cdot a_{twist}}$$

$$M_t \cdot a_{twist} := \frac{(F_{bu} - F_{bi}) \cdot \frac{1}{2} \cdot b}{2 \cdot \pi} - M_{obu} \cdot R_{bu} - M_{obi} \cdot R_{bi}$$

According (IX) we find the angular displacement of the twisting ring:

$$\theta := 12 \cdot \frac{\frac{(F_{bu} - F_{bi}) \cdot b}{4 \cdot \pi} - M_{obu} \cdot R_{bu} - M_{obi} \cdot R_{bi}}{\gamma}$$

In this case an overpressure exists from the left of the twisting ring to the right of the twisting ring.
Since the twisting ring must be in axial direction be in balance we can say:

$$F_{bi} + F_{bu} := p_{bu} \cdot \pi \cdot \left(R_{i\_bu}^2 - R_{o\_bi}^2\right) \qquad F_{bi} - F_{bu} := P_{bu} \qquad \text{(XI)}$$

or

$$F_{bu} := P_{bu} - F_{bi} \qquad \text{(XIa)}$$

$R_{i\ bu}$ is the inner radius of the jacket pipe and $R_{o\ bi}$ is the outer radius of the core pipe.
The angular displacement than is:

$$\theta := 12 \cdot \frac{\frac{(P_{bu} - 2 \cdot F_{bi}) \cdot b}{4 \cdot \pi} - M_{obu} \cdot R_{bu} - M_{obi} \cdot R_{bi}}{\gamma} \qquad \text{(XII)}$$

Equal displacements of both cylinders

The moment couples $M_{o\,bu}$ and $M_{o\,bi}$ are found from the boundary conditions of dw/dx and w of the cylinders and the equity of the twisting ring

If we neglegt the stretching of the twisting ring by the radial forces the following is true:

$$\omega_{bu} := 0 \qquad \omega_{bi} := 0 \qquad \text{(XIV)}$$

The jacket pipe has a free-end, the core pipe is continues, so from (279) and (VII) we determine the following:

$$\omega_{bu} := \frac{-1\cdot\left(\beta_{bu}\cdot M_{obu} + Q_{obu}\right)}{2\cdot\beta_{bu}^{3}\cdot D_{bu}} + \omega_{bu}(p) \qquad \text{(XV)}$$

$$\omega_{bi} := -\frac{Q_{obi}}{8\cdot\beta_{bi}^{3}\cdot D_{bi}} + \frac{1}{2}\cdot\left(\omega_{rbi}(p) + \omega_{lbi}(p)\right) \qquad \text{(XVI)}$$

From equation (XIV) thru (XVI) we derive:

$$\beta_{bu}\cdot M_{obu} + Q_{obu} := 2\cdot\beta_{bu}^{3}\cdot D_{bu}\cdot\omega_{bu}(p)$$

$$Q_{obu} := -\beta_{bu}\cdot M_{obu} + 2\cdot\beta_{bu}^{3}\cdot D_{bu}\cdot\omega_{bu}(p) \qquad \text{(XVII)}$$

$$Q_{obi} := 8\cdot\beta_{bi}^{3}\cdot D_{bi}\cdot\frac{1}{2}\cdot\left(\omega_{rbi}(p) + \omega_{lbi}(p)\right)$$

$$Q_{obi} := 4\cdot\beta_{bi}^{3}\cdot D_{bi}\cdot\left(\omega_{rbi}(p) + \omega_{lbi}(p)\right) \qquad \text{(XVIII)}$$

Equal angular displacement of both cylinders

The rotations according (280) and (VIII) of both cylinders at the location of the twisting ring are identical:

$$\left(\frac{dw}{dx}\right)_{bu} := \left(\frac{dw}{dx}\right)_{lbi}$$

$$\frac{2\cdot\beta_{bu}\cdot M_{obu} + Q_{bu}}{2\cdot\beta^{2}\cdot D_{bu}} := \frac{M_{obi}}{4\cdot\beta_{bi}\cdot D_{bi}} + \frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

By substituting (XVII) we find:

$$\frac{2\cdot\beta_{bu}\cdot M_{obu} - \beta_{bu}\cdot M_{obu} + 2\cdot\beta_{bu}^{3}\cdot D_{bu}\cdot\omega_{bu}(p)}{2\cdot\beta_{bu}^{2}\cdot D_{bu}} := \frac{M_{obi}}{4\cdot\beta_{bi}\cdot D_{bi}} + \frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

$$\beta_{bu}\cdot M_{obu} + 2\cdot\beta_{bu}^{3}\cdot D_{bu}\cdot\omega_{bu}(p) := 2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\left(\frac{M_{obi}}{4\cdot\beta_{bi}\cdot D_{bi}} + \frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right)$$

$$\beta_{bu}\cdot M_{obu} := -2\cdot\beta_{bu}^{3}\cdot D_{bu}\cdot\omega_{bu}(p) + \frac{\beta_{bu}^{2}\cdot D_{bu}\cdot M_{obi}}{2\cdot\beta_{bi}\cdot D_{bi}} + \beta_{bu}^{2}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

$$M_{obu} := -2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p) + \frac{\beta_{bu}\cdot D_{bu}\cdot M_{obi}}{2\cdot\beta_{bi}\cdot D_{bi}} + \beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

$$M_{obu} := M_{obi}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bi}\cdot D_{bi}} - 2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p) + \beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi} \qquad \text{(XX)}$$

The resulting angular displacement
according (XII) we find:

$$\theta := 12\cdot\frac{\frac{\left(P_{bu}-2\cdot F_{bi}\right)\cdot b}{4\cdot\pi}-M_{obu}\cdot R_{bu}-M_{obi}\cdot R_{bi}}{\gamma}$$

With (XX) we find:

$$\theta := \frac{12\left[\frac{\left(P_{bu}-2\cdot F_{bi}\right)\cdot b}{4\cdot\pi}-M_{obi}\cdot R_{bi}-\blacksquare \ldots \\ +R_{bu}\cdot\left(M_{obi}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bi}\cdot D_{bi}}-2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p)+\beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right)\right]}{\gamma}$$

According (VIII) also the following is true:

$$\theta := \left(\frac{dw}{dx}\right)_{lbi} \qquad \theta := \frac{M_{obi}}{4\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

We substititue this into the formula found

$$\frac{12\left[\frac{\left(P_{bu}-2\cdot F_{bi}\right)\cdot b}{4\cdot\pi}-\blacksquare \ldots \\ +M_{obi}\cdot R_{bi}-\blacksquare \ldots \\ +R_{bu}\cdot\left[M_{obi}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bi}\cdot D_{bi}}-\blacksquare \ldots \\ +2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p)+\blacksquare \ldots \\ +\beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right]\right]}{\gamma} := \frac{M_{obi}}{4\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

==

$$\frac{\left(P_{bu}-2\cdot F_{bi}\right)\cdot b}{4\cdot\pi}-\blacksquare \ldots \\ +M_{obi}\cdot R_{bi}-\blacksquare \ldots \\ +R_{bu}\cdot\left[M_{obi}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bi}\cdot D_{bi}}-\blacksquare \ldots \\ +2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p)+\blacksquare \ldots \\ +\beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right] := \frac{\gamma\cdot M_{obi}}{48\cdot\beta_{bi}\cdot D_{bi}}+\frac{\gamma\cdot\beta_{bi}\cdot\Delta\omega_{bi}}{24}$$

==

$$\frac{\left(P_{bu}-2\cdot F_{bi}\right)\cdot b}{4\cdot\pi} := R_{bu}\cdot\left(M_{obi}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bi}\cdot D_{bi}}-2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p)+\beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right)+\blacksquare \ldots \\ +M_{pbi}\cdot R_{bi}+\frac{\gamma\cdot M_{obi}}{48\cdot\beta_{bi}\cdot D_{bi}}+\frac{\gamma\cdot\beta_{bi}\cdot\Delta\omega_{bi}}{24}$$

==

$$\frac{(P_{bu}-2\cdot F_{bi})\cdot b}{4\cdot\pi} := M_{obi}\cdot\left(R_{bu}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bi}\cdot D_{bi}}+R_{bi}+\frac{\gamma}{48\cdot\beta_{bi}\cdot D_{bi}}\right)+\ \ldots$$
$$+R_{bu}\cdot\left(-\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p)+\beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right)+\ \ldots$$
$$+\frac{\gamma\cdot\beta_{bi}\cdot\Delta\omega_{bi}}{24}$$

With the following abbreviations:

$$\rho_1 := R_{bu}\cdot\frac{\beta_{bu}\cdot D_{bu}}{2\cdot\beta_{bu}\cdot D_{bu}}+R_{bi}+\frac{\gamma}{48\cdot\beta_{bi}\cdot D_{bi}} \qquad \text{(XXIa)}$$

$$\rho_2 := R_{bu}\cdot\left(-2\cdot\beta_{bu}^{2}\cdot D_{bu}\cdot\omega_{bu}(p)+\beta_{bu}\cdot D_{bu}\cdot\beta_{bi}\cdot\Delta\omega_{bi}\right)+\frac{\gamma\cdot\beta_{bi}\cdot\Delta\omega_{bi}}{24} \qquad \text{(XXIb)}$$

We can write the equation like this:

$$\frac{(P_{bu}-2\cdot F_{bi})\cdot b}{4\cdot\pi} := \rho_1\cdot M_{obi}+\rho_2$$

This results in:

$$M_{obi} := \frac{(P_{bu}-2\cdot F_{bi})\cdot b}{4\cdot\pi\cdot\rho_1}-\frac{\rho_2}{\rho_1} \qquad \text{(XXII)}$$

With (VIII) we find:

$$\theta := \left(\frac{dw}{dx}\right)_{lbi} \qquad \theta := \frac{M_{obi}}{4\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

$$\theta := \frac{\dfrac{(P_{bu}-2\cdot F_{bi})\cdot b}{4\cdot\pi\cdot\rho_1}-\dfrac{\rho_2}{\rho_1}}{4\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

$$\theta := \frac{(P_{bu}-2\cdot F_{bi})\cdot b}{16\cdot\pi\cdot\rho_1\cdot\beta_{bi}\cdot D_{bi}}-\frac{\dfrac{\rho_2}{\rho_1}}{4\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi}$$

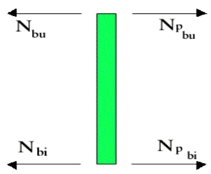
$$\theta := \frac{(P_{bu}-2\cdot F_{bi})\cdot b}{16\cdot\pi\cdot\rho_1\cdot\beta_{bi}\cdot D_{bi}}-\frac{\rho_2}{4\cdot\rho_1\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot\beta_{bi}\cdot\Delta\omega_{bi} \qquad \text{(XXIII)}$$

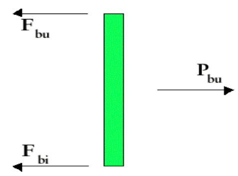
With this equation we can determine the **relative axial displacement** between the two end of the twisting ring:

$$\Delta_{ax} := \theta\cdot b$$

$$\Delta_{ax} := \frac{(P_{bu}-2\cdot F_{bi})\cdot b^2}{16\cdot\pi\cdot\rho_1\cdot\beta_{bi}\cdot D_{bi}}-\frac{b\cdot\rho_2}{4\cdot\rho_1\cdot\beta_{bi}\cdot D_{bi}}+\frac{1}{2}\cdot b\cdot\beta_{bi}\cdot\Delta\omega_{bi} \qquad \text{(XXIV)}$$

Distrubution and transfer of forces between inner pipe, outer pipe and twisting ring.





At the right side of the twisting ring a nominal force N_{pbi} [N] acts on the inner pipe. At the left side of the ring a nominal force N_{bi} acts on the inner pipe.
The reaction forces F_{bi} and F_{bu} of the inner and outer pipe act on the twisting ring. These reaction forces can be expressed as:

$F_{bi} := N_{bi} - N_{pbi}$ or $N_{bi} := N_{pbi} + F_{bi}$ (XXV)

A similar approach can be used for the outer pipe:

$F_{bu} := N_{bu} - N_{pbu}$ or $N_{bu} := N_{pbu} + F_{bu}$ (XXVI)

We use formula (XIa): $F_{bu} := P_{bu} - F_{bi}$

Substitution of (XIa) in (XXVI) results in: $N_{bu} := N_{pbu} + P_{bu} - F_{bi}$ (XXVII)

Axial displacements of both cylinders
The expansion at temperature rise delta T_{bi} of the inner cylinder is:

$$\Delta l_{bi} := \varepsilon_{axbi} \cdot l_{bi} + \alpha_{bi} \cdot l_{bi} \cdot \Delta T_{bi}$$

$$\Delta l_{bi} := \left(\frac{\sigma_{axbi}}{E_{bi}} - \nu \cdot \frac{\sigma_{tgbi}}{E_{bi}} \right) \cdot l_{bi} + \alpha_{bi} \cdot l_{bi} \cdot \Delta T_{bi}$$

$$\Delta l_{bi} := \left(\frac{N_{bi}}{E_{bi} \cdot A_{bi}} - \nu \cdot \frac{\sigma_{tgbi}}{E_{bi}} \right) \cdot l_{bi} + \alpha_{bi} \cdot l_{bi} \cdot \Delta T_{bi}$$

A_{bi} is the cross sectional area of the inner cylinder.

$$\Delta l_{bi} := \frac{N_{bi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} - \frac{\nu \cdot l_{bi} \cdot \sigma_{tgbi}}{E_{bi}} + \alpha_{bi} \cdot l_{bi} \cdot \Delta T_{bi}$$

Similar holds true for the outer cylinder:

$$\Delta l_{bu} := \frac{N_{bu} \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \frac{\nu \cdot l_{bu} \cdot \sigma_{tgbu}}{E_{bu}} + \alpha_{bu} \cdot l_{bu} \cdot \Delta T_{bu}$$

We use the following abbreviations:

$$\mu_{bi} := \frac{-\nu \cdot l_{bi} \cdot \sigma_{tgbi}}{E_{bi}} + \alpha_{bi} \cdot l_{bi} \cdot \Delta T_{bi} \qquad \mu_{bu} := \frac{-\nu \cdot l_{bu} \cdot \sigma_{tgbu}}{E_{bu}} + \alpha_{bu} \cdot l_{bu} \cdot \Delta T_{bu}$$

We substitute these abbreviations in the previous equations, than we find:

$$\Delta l_{bi} := \frac{N_{bi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} \qquad \Delta l_{bu} := \frac{N_{bu} \cdot l_{bu}}{E_{bu} \cdot A_{bu}} + \mu_{bu} \qquad \text{(XXVIII)}$$

TWISTING OF A CIRCULAR RING

CALCULATION OF LOCAL STRESSES

Date: Di 01-Jan-2002
Time: 18:55:46

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With the angular displacement of the twisting ring we find:

$$\Delta_{ax} := \Delta l_{bi} - \Delta l_{bu}$$

$$\Delta_{ax} := \frac{N_{bi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} - \left(\frac{N_{bu} \cdot l_{bu}}{E_{bu} \cdot A_{bu}} + \mu_{bu} \right) \qquad \text{(XXIX)}$$

We use equation (XXV) and (XXVII):

$$N_{bi} := N_{pbi} + F_{bi} \qquad N_{bu} := N_{pbu} + P_{bu} - F_{bi}$$

We substitude these in (XXIX), then we find:

$$\Delta_{ax} := \frac{(N_{pbi} + F_{bi}) \cdot l_{bi}}{E_{bi} \cdot A_{.bi}} + \mu_{bi} - \frac{(N_{pbu} + P_{bu} - F_{bi}) \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \mu_{bu} \qquad \text{(XXX)}$$

Substitution of equations of axial displacement

We use equation (XXIV):

$$\Delta_{ax} := \theta \cdot b \qquad \Delta_{ax} := \frac{(P_{bu} - 2 \cdot F_{bi}) \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} - \frac{b \cdot \rho_2}{4 \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} + \frac{1}{2} \cdot b \cdot \beta_{bi} \cdot \Delta\omega_{bi}$$

Substitution of (XXIV) in (XXX) gives:

$$\frac{(P_{bu} - 2 \cdot F_{bi}) \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} - \frac{b \cdot \rho_2}{4 \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} + \frac{1}{2} \cdot b \cdot \beta_{bi} \cdot \Delta\omega_{bi} := \frac{(N_{pbi} + F_{bi}) \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} \dots$$
$$+ - \frac{(N_{pbu} + P_{bu} - F_{bi}) \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \mu_{bu}$$

From this we can find the following:

$$F_{bi} \cdot \left(-\frac{l_{bu}}{E_{bu} \cdot A_{bu}} - \frac{2 \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} - \frac{l_{bi}}{E_{bi} \cdot A_{bi}} \right) := \frac{N_{pbi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} - \dots$$
$$+ \frac{(N_{pbu} + P_{bu}) \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \mu_{bu} - \dots$$
$$+ \frac{P_{bu} \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} + \frac{b \cdot \rho_2}{4 \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} - \dots$$
$$+ \frac{1}{2} \cdot b \cdot \beta_{bi} \cdot \Delta\omega_{bi}$$

From this equation we can calculate F_{bi}

$$F_{bi} = \frac{\frac{N_{pbi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} - \frac{(N_{pbu} + P_{bu}) \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \mu_{bu} - \frac{P_{bu} \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} + \frac{P_{bu} \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} + \frac{b \cdot \rho_2}{4 \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} - \frac{b \cdot \beta_{bi} \cdot \delta\omega_{bi}}{2}}{\frac{-l_{bi}}{E_{bi} \cdot A_{bi}} - \frac{2 \cdot b^2}{16 \cdot \pi \cdot \rho_1 \cdot \beta_{bi} \cdot D_{bi}} - \frac{l_{bu}}{E_{bu} \cdot A_{bu}}}$$

We calculate the other forces and moment couples
with the following equations:

P_{bu} XI
N_{bi} XXV
N_{bu} XXVII
F_{bu} XXVI
M_{obi} XXII

θ VIIIbi
M_{obu} XX
Q_{obu} XVII
Q_{obi} XVIII
$\beta_{bi} \cdot M_{olbi}$ VI
Q_{olbi} V
Q_{orbi} IIa
M_{orbi} Ia

$$\mu_{bi} := \frac{-\nu \cdot l_{bi} \cdot \sigma_{tgbi}}{E_{bi}} + \alpha_{bi} \cdot l_{bi} \cdot \Delta T_{bi}$$

$$\mu_{bu} := \frac{-\nu \cdot l_{bu} \cdot \sigma_{tgbu}}{E_{bu}} + \alpha_{bu} \cdot l_{bu} \cdot \Delta T_{bu}$$

Calculation of stresses

The stresses in the twisting ring are calculated according lit. 2 (129):

$$\sigma_{max\_ring} := \frac{6 \cdot M_t \cdot a_{twist}}{h^2 \cdot c \cdot \ln\left(\frac{d}{c}\right)}$$

The stresses in the cylinders are calculated according lit. 1

$$S1_{bi} := -0.5 \cdot p_{bi} \qquad S1_{bu} := -0.5 \cdot p_{bu}$$

$$S2_{bi} := \sigma_{tgbi} \qquad S2_{bu} := \sigma_{tgbu}$$

$$S3_{rbi} := \sigma_{axbi} + \sigma_{rbi} \qquad S3_{bu} := \sigma_{axbu} + \sigma_{bu}$$

$$S3_{lbi} := \sigma_{axbi} + \sigma_{lbi}$$

$$S3_{rbi} := \text{if}\left(\left|\sigma_{axbi} - \sigma_{rbi}\right| > \left|S3_{rbi}\right|, \sigma_{axbi} - \sigma_{rbi}, S3_{rbi}\right)$$

$$S3_{lbi} := \text{if}\left(\left|\sigma_{axbi} - \sigma_{lbi}\right| > \left|S3_{lbi}\right|, \sigma_{axbi} - \sigma_{lbi}, S3_{lbi}\right)$$

$$S3_{bu} := \text{if}\left(\left|\sigma_{axbu} - \sigma_{bu}\right| > \left|S3_{bu}\right|, \sigma_{axbu} - \sigma_{bu}, S3_{bu}\right)$$

$$\sigma_{eqrbi} := \sqrt{0.5 \cdot \left[\left(S1_{bi} - S2_{bi}\right)^2 + \left(S2_{bi} - S3_{rbi}\right)^2 + \left(S3_{rbi} - S1_{bi}\right)^2\right]}$$

$$\sigma_{eqlbi} := \sqrt{0.5 \cdot \left[\left(S1_{bi} - S2_{bi}\right)^2 + \left(S2_{bi} - S3_{lbi}\right)^2 + \left(S3_{lbi} - S1_{bi}\right)^2\right]}$$

$$\sigma_{eqbu} := \sqrt{0.5 \cdot \left[\left(S1_{bu} - S2_{bu}\right)^2 + \left(S2_{bu} - S3_{bu}\right)^2 + \left(S3_{bu} - S1_{bu}\right)^2\right]}$$