DISTRIBUTION OF SHEAR STRESSES IN PIPE

CASE STUDY ON LOCAL SHEAR STRESS DETERMINATION

Date: Za 09-Feb-2002

Time: 10:07:55

Project: IMPROVEMENTS OF FUTURE DEVELOPMENTS

Jobnr: PV2002

ABSTRACT:

The shear stress resulting from a lateral shear force in the pipeline is normally assumed to variate according a sinus curve over the circumference. Moreover the shear stress is assumed to be uniformly distributed over the wall thickness.

In this case study we will proof that both assumptions are not correct.

We will show that the deviation exists mainly at the top and bottom of the cross section of the pipe relative to the neutral line.

It has not been my intension to abuse any person or company with these small articles and case studies. The statements in these studies are for discussion reference only and you are kindly invited to respond.

All positive contributions are welcome.

REFERENCES:

Formulas for Stress and Strain 5th Edition R.J. Roark / W.C. Young
Berechnung und konstruction ringversteifter Druckrohrleitungen F. Mang 1966
Strenght of materials G. Rumpel and H.D. Sondershausen (Dubbel)
Technische Formules K.Gieck 1969
Technical reference guide PLE
Technical reference guide P10
Technical reference guide Autopipe
Technical reference guide Caesar II

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Compairing differend methods of calculating the local stresses in a pipe

Input data

Outside diameter		$D_o := 508 \cdot mm$
Wallthickness		$d_d := 11 \cdot mm$
Design pressure		$P_d := 80 \cdot bar$
Modulus of elasticity		$E := 205800 \cdot \frac{N}{mm^2}$
Shear force		$F_{lat} := 1000 \cdot N$
Bending moment		$M_b := 100000 \cdot N \cdot m$
Torsional moment		$M_t := 1000 \cdot N \cdot m$

Calculated data

Inside diameter	$D_i := D_o - 2 \cdot d_d$	$D_i = 486 \cdot mm$
Outside radius pipe wall	$R_u := 0.5 \cdot D_o$	$R_u = 254 \cdot mm$
Inside radius of pipe wall	$R_i := 0.5 \cdot D_i$	$R_i = 243 \cdot mm$
Average diameter	$D_g := D_o - d_d$	$D_g = 497 \cdot mm$
Cross sectional area pipe	$A_{pipe} := \frac{\pi}{4} \cdot \left(D_o^2 - D_i^2\right)$	$A_{pipe} = 17175.087 \cdot mm^2$
Moment of inertia pipe	$I_{pipe} := \frac{\pi}{64} \cdot \left(D_o^4 - D_i^4\right)$	$I_{pipe} = 5.306 \cdot 10^8 \cdot mm^4$

φ is the angle of the Lateral shear force vector relative to the z-axis $\phi := 90 \cdot deg$

α is the angle under consideration and varies from 0 degrees to 360 degrees

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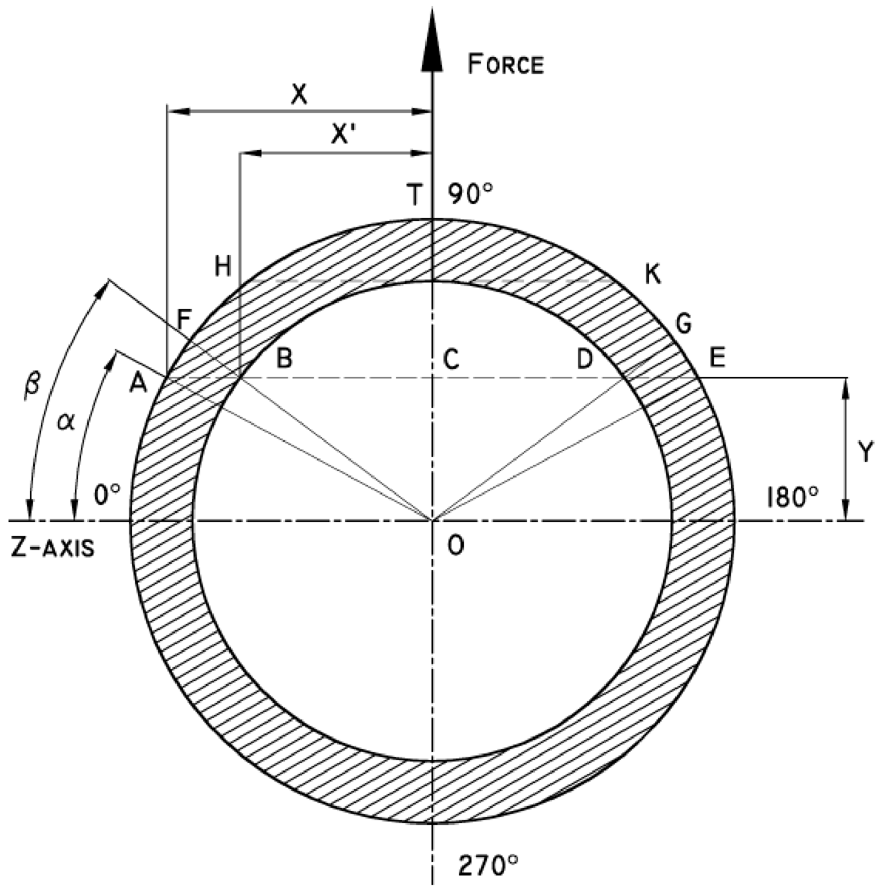
The formula's used in the comercial computer programmes are based on thin cylindrical shells. In this case study we will proof that the shear stress does NOT simply variate according sin(α) over the circumference. We will proof that there is a discontinuity in the area above line H-K in the next figure. We will use the general equations of calculating shear in the pipe model.



At first we determine the length of the lenght A - B and D - E. These lenght vary over the circumference. We have worked with the absolute value only. Later on we include a sign convention of the shear force stresses.

Equation of A -C

$$x(D,\alpha) := 0.5 \cdot D \cdot \cos(\alpha)$$

Equation of O - C

$$y(D,\alpha) := 0.5 \cdot D \cdot \sin(\alpha)$$

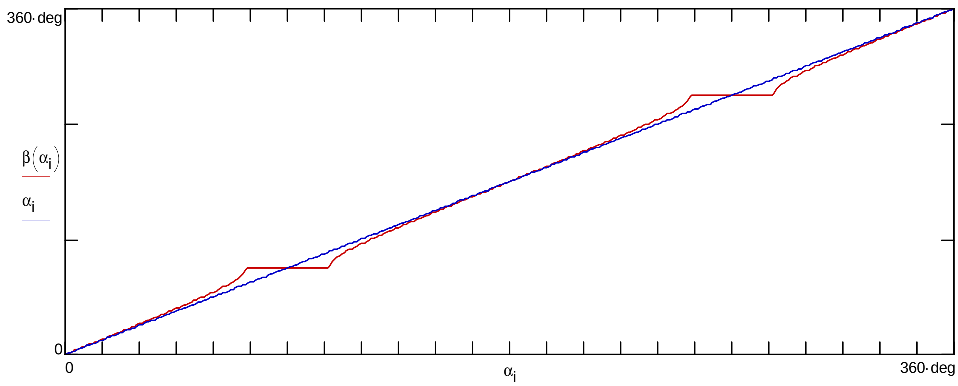
Equation of B - C

$$x'(\alpha) := 0.5 \cdot D_i \cdot \sin\left(\operatorname{acos}\left(\frac{y(D_o,\alpha)}{0.5 \cdot D_i}\right)\right)$$

Equation of the variation over the circumference of point 'B'

$$\beta(\alpha) := \mathrm{if}\left(\left|y(D_o,\alpha)\right| \le 0.5 \cdot D_i, \left|\operatorname{asin}\left(\frac{D_o \cdot \left|\sin(\alpha)\right|}{D_i}\right)\right|, 90 \cdot \mathrm{deg}\right)$$

Graph showing β as a function of α. As α grows point "A" will pass "H". At that point β is 90 deg and remains so until point "A" passes point "K".



LOCAL SHEAR STRESS RESULTING FROM A LATERAL FORCE IN THE CROSS SECTION OF A PIPE

Some pipestress programmes (like PLE) calculate the shear stress from a lateral force according the following equation

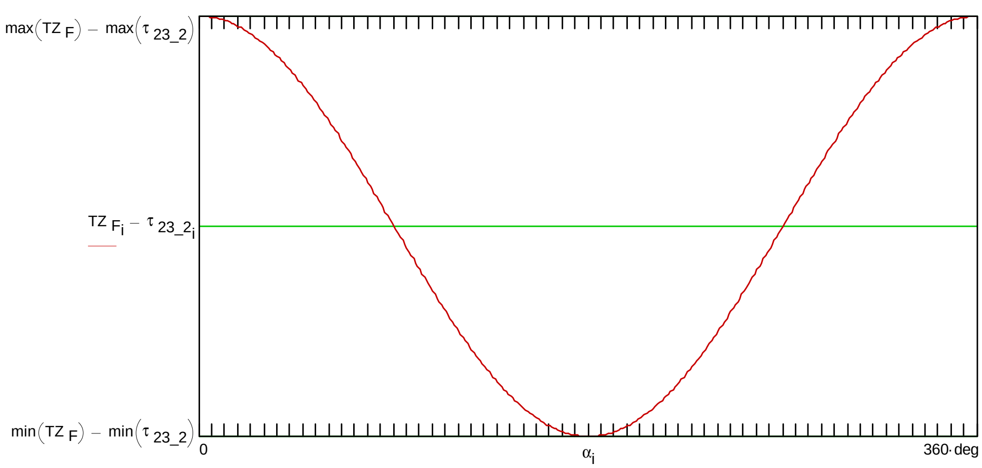
$$TZ_{F_i} := -2 \cdot \frac{F_{lat}}{A_{pipe}} \cdot \sin\left(\alpha_i - \phi\right)$$

Some other programmes (like P10) use a more complex equation

$$\eta := \frac{2 \cdot D_g^{\,2}}{D_g^{\,2} + d_d^{\,2}} \qquad \tau_{23\_2_i} := \frac{-\eta \cdot F_{lat}}{A_{pipe}} \cdot \sin\left(\alpha_i - \phi\right)$$

Both equations give approximately the same results. And both equations variate according a sinus curve over the circumference. In this case study we will use the general equation and will proof that there is a discontinuity of the stresses on the top and bottom of the cross section.

Graph showing the small deviation between the two widely used equations



$$\max\left(TZ_F\right) - \max\left(\tau_{23\_2}\right) = 5.702 \cdot 10^{-5} \cdot MPa \qquad \min\left(TZ_F\right) - \min\left(\tau_{23\_2}\right) = -5.702 \cdot 10^{-5} \cdot MPa$$

Calculation of the cross sectional area above line A-E

A1 is area of segment A-T-E

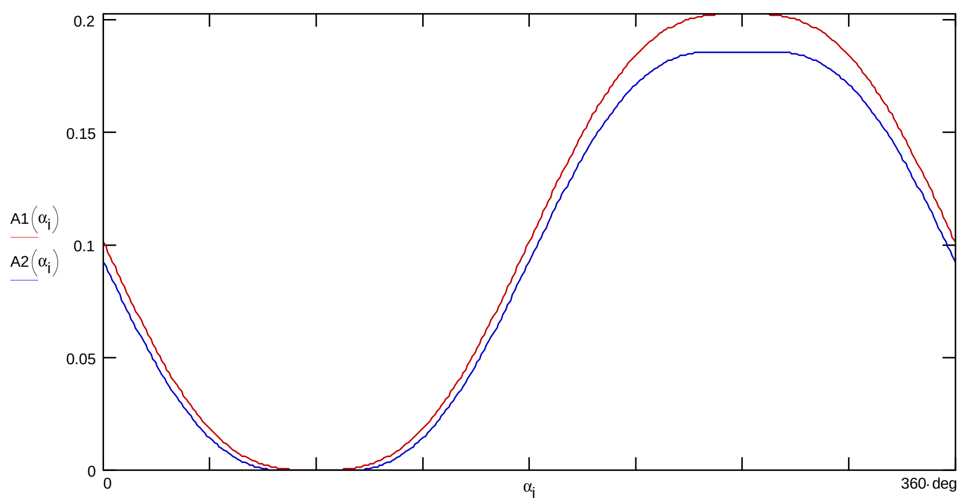
$$A1(\alpha) := \left| 2\cdot\frac{\operatorname{atan}\left(\frac{x\left(D_o,\alpha\right)}{y\left(D_o,\alpha\right)}\right)}{360\cdot deg}\cdot\frac{\pi}{4}\cdot D_o^{\,2} - x\left(D_o,\alpha\right)\cdot y\left(D_o,\alpha\right)\right|$$

A2 is area of segment B-D

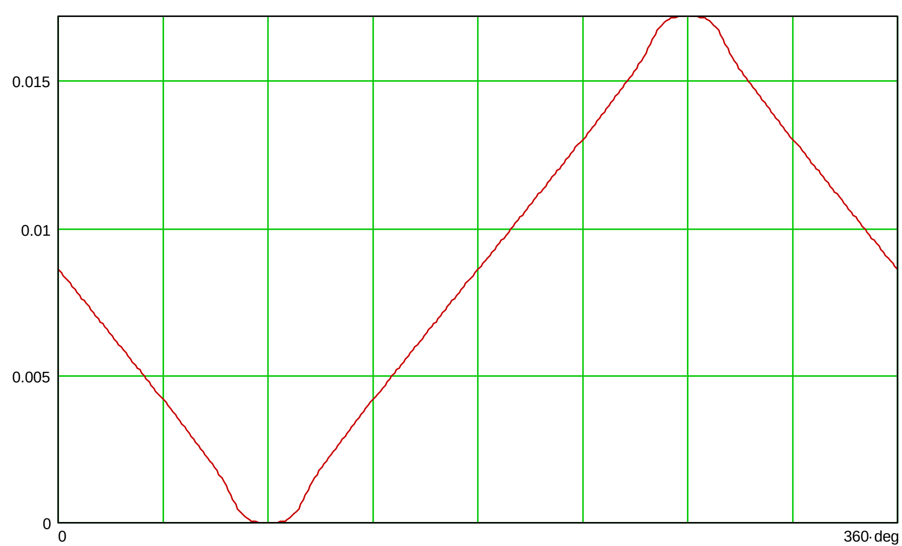
$$A2(\alpha) := \operatorname{if}\left(\left|y\left(D_o,\alpha\right)\right| \le \frac{D_i}{2}, \left|2\cdot\frac{\operatorname{atan}\left(\frac{x'(\alpha)}{y\left(D_o,\alpha\right)}\right)}{360\cdot deg}\cdot\frac{\pi}{4}\cdot D_i^{\,2} - x'(\alpha)\cdot y\left(D_o,\alpha\right)\right|, 0\cdot mm^2\right)$$

The area of the cross section above the line A -E of the is found by subtraction

$$A(\alpha) := A1(\alpha) - A2(\alpha)$$



Graph of Cross sectional area above line A - E



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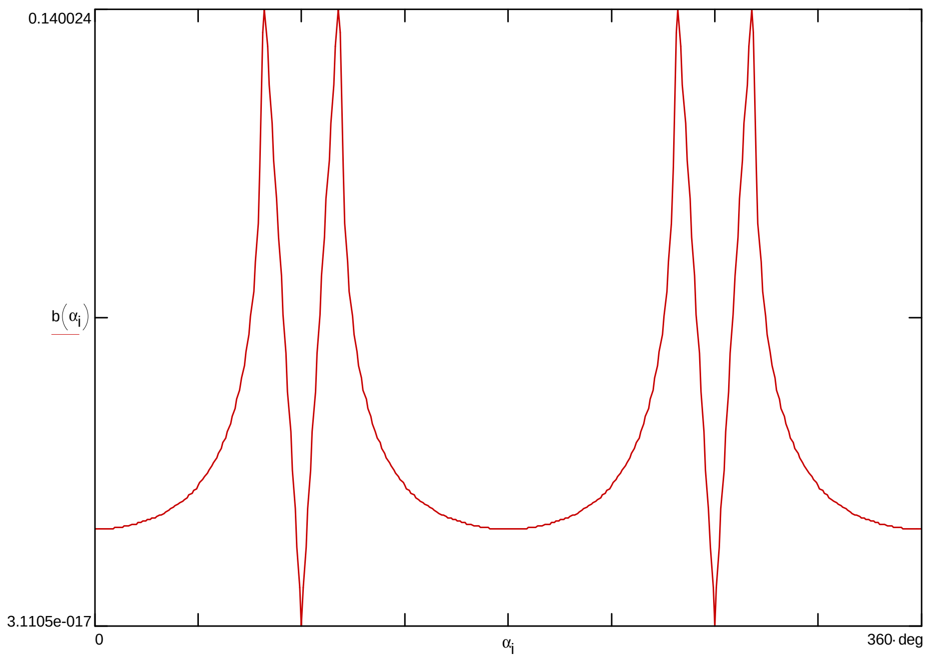
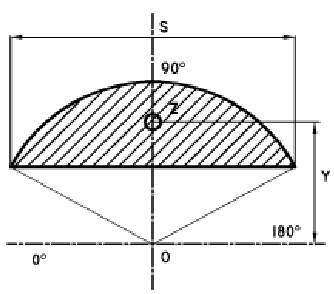
Length of line A-B and D-E at location "C". This is the line under consideration. It may be clear that the length is not constant.

$$b(\alpha) := \left(\left| \cos(\alpha) \cdot \frac{D_o}{2} \right| - \left| \cos\left(\operatorname{asin}\left(\frac{|0.5 \cdot D_o \cdot \sin(\alpha)|}{0.5 \cdot D_i} \right) \right) \cdot \frac{D_i}{2} \right| \right) \cdot 2$$

$$b(\alpha) := \operatorname{if}\left(|0.5 \cdot D_o \cdot \sin(\alpha)| \geq 0.5 \cdot D_i, |\cos(\alpha) \cdot D_o|, b(\alpha) \right)$$



Calculation of loadpoint from cross section of the segment of a circle



$$y := \frac{s^3}{12 \cdot A}$$

$$z1(\alpha) := \frac{\left(2 \cdot |x(D_o, \alpha)|\right)^3}{12 \cdot A1(\alpha)}$$

$$z2(\alpha) := \frac{\left(2 \cdot |x'(\alpha)|\right)^3}{12 \cdot A2(\alpha)}$$

Loadpoint of pipe cross section above line A - E

$$z(\alpha) := \frac{z1(\alpha) \cdot A1(\alpha) - z2(\alpha) \cdot A2(\alpha)}{A(\alpha)}$$

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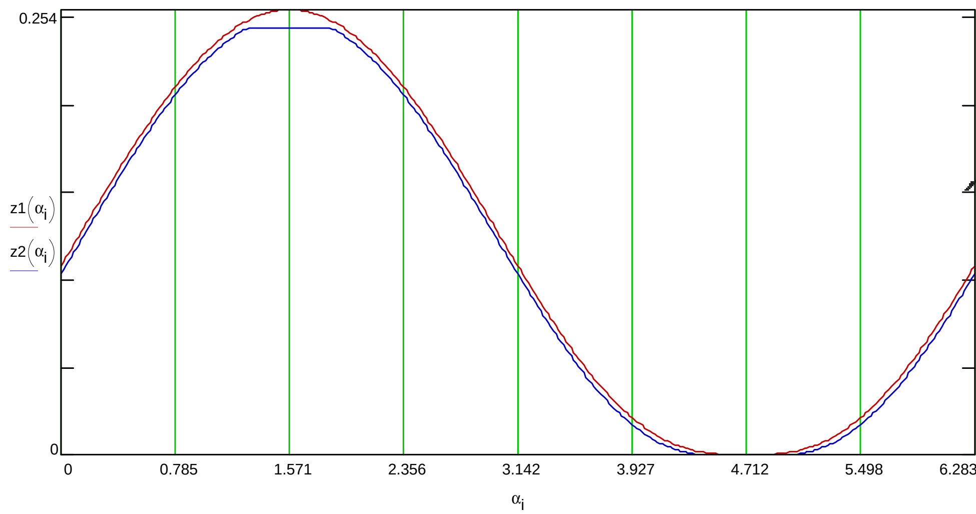
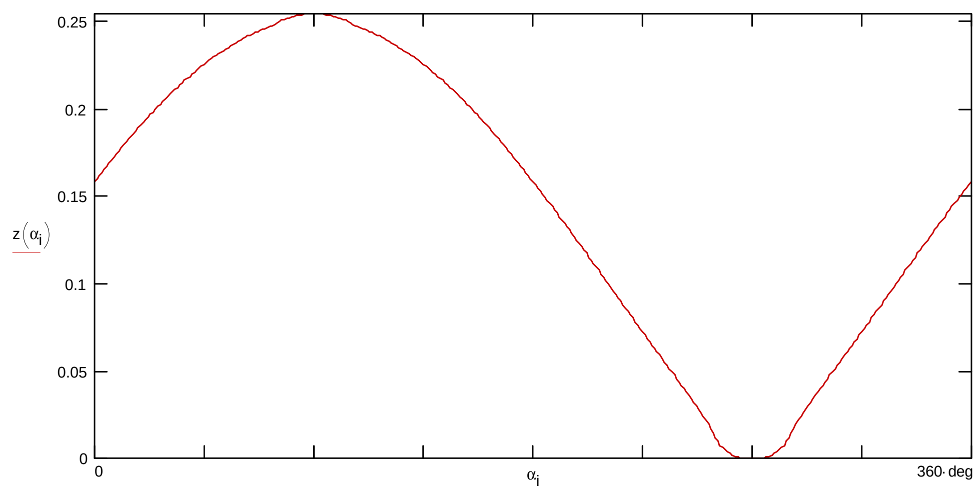
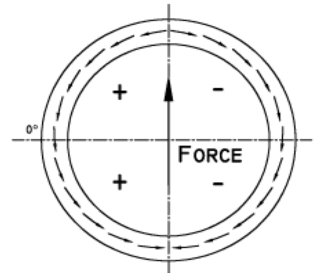




In this case study we have worked with a z that is positive for the whole circumference. Since all variables of the equation of the shear stress are positive this would mean that no negative stress would occur that is not the case. The stress is negative in the second and third quadrant.

SIGN CONVENTION OF REACTION FORCES



z is adjusted according this sign convention

$$z(\alpha) := z(\alpha)\cdot\frac{|\cos(\alpha)|}{\cos(\alpha)}$$

The shear stress at any point according Formulas for Stress and Strain page 91 formula (2)

$$\tau(\alpha) := \frac{F_{lat}\cdot A(\alpha)\cdot z(\alpha)}{I_{pipe}\cdot b(\alpha)} \qquad \tau_{inside}(\alpha) := \tau(\beta(\alpha))$$

$TZ_{F_{180}} = -0.116\cdot MPa$ ***<------ Check ------>*** $\tau(\alpha_{180}) = -0.116\cdot MPa$

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Graph showing the discontinuities at the top and bottom of the pipe relative to the neutral line. It also shows a difference between the outer and inner pipe and thus proofing that the shear stress is not uniformly distributed over the wall thickness.