CIRCULAR RING IN JACKETED SYSTEM

CALCULATION OF THE SPRINGRATE AT THE CONNECTION

Date: Zo 07-Feb-2021
Time: 10:56:49

Project: VARIOUS CALCULATIONS
Jobnr: PV2021

ABSTRACT:

Calculation of stresses of a circular ring attached to a pipe according to
(Lit. 1) S.Timoshenko Strength of materials part II 3rd edition page 140 143 par 28 and
(Lit. 2) according to S.Timoshenko Theory of Plates and Shells 2nd edition page 468 469.

This theory is used to calculate the stresses in the inner pipe due to a plate welded in a jacketed system between inner and outer pipe.

Moreover a springrate is calculated of the connection to be used in a pipestress programme. This spring allows some relative displacement of the inner pipe versus the outer pipe.

REFERENCES:

T. Muilman Mui970210 Rev.A 10 maart 1997
S.Timoshenko Strength of materials part II 3rd edition page 140-143 par.28
S.Timoshenko Theory of Plates and Shells 2nd edition page 468-469

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Calculation of stresses of a circular ring attached to a pipe according to (Lit. 1) S.Timoshenko 'Strength of materials part II' 3rd edition page 140–143 par 28 and (Lit. 2) according to S.Timoshenko 'Theory of Plates and Shells' 2nd edition page 468–469.

This theory is used to calculate the stresses in the inner pipe due to a plate welded in a jacketed system between inner and outer pipe.

Outside diameter pipe: $OD_{bi} = 12.75 \cdot in$

Wallthickness pipe : $t_{bi} = 0.375 \cdot in$

Pressure pipe : $p_{bi} = 11.4 \cdot bar$

Expansion pipe : $\alpha_{bi} = 2.223 \cdot \frac{mm}{m}$

Modulus of elasticity pipe: $E_{bi} = 186738.182 \cdot MPa$

Length of jacket : $L_{bu} = 4910 \cdot mm + 2 \cdot 100 \cdot mm$

Outside diameter jacket: $OD_{bu} = 16 \cdot in$

Wallthickness jacket : $t_{bu} = 0.165 \cdot in$

Pressure jacket : $p_{bu} = 10.5 \cdot bar$

Expansion jacket : $\alpha_{bu} = 2.7552 \cdot \frac{mm}{m}$

Modulus of elasticity jacket : $E_{bu} = 184883.64 \cdot MPa$

Plate thickness : $t_{plate} = 12 \cdot mm$

Poisson ratio : $\mu = 0.303$

Modulus of elasticity: $E_{plate} = 184883.64 \cdot MPa$

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From note Mui970210 we derive the following formula:

$$F_{bi} \cdot \left[\frac{-l_{bi}}{E_{bi} \cdot A_{bi}} - \frac{2 \cdot b^2}{16 \cdot \pi \cdot \rho \cdot \beta_{bi} \cdot D_{bi}} - \frac{l_{bu}}{E_{bu} \cdot A_{bu}} \right] = \square$$

$$\frac{N_{pbi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} - \frac{\left[N_{pbu} + P_{bu}\right] \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \mu_{bu} - \frac{P_{bu} \cdot b^2}{16 \cdot \pi \cdot \rho \cdot \beta_{bi} \cdot D_{bi}} \square$$

$l_{bu} = L_{bu} \cdot 0.5$ $\qquad$ $l_{bi} = l_{bu}$

$R_{ibu} = \left[OD_{bu} - t_{bu}\right] \cdot 0.5$ $\qquad$ $R_{obi} = \left[OD_{bi}\right] \cdot 0.5$

$P_{bu} = p_{bu} \cdot \pi \cdot \left[R_{ibu}^2 - R_{obi}^2\right]$ $\qquad$ $P_{bu} = 46918.053 \cdot N$

$A_{bi} = \pi \cdot t_{bi} \cdot \left[OD_{bi} - 2 \cdot t_{bi}\right]$ $\qquad$ $A_{bu} = \pi \cdot t_{bu} \cdot \left[OD_{bu} - 2 \cdot t_{bu}\right]$

We assume that the plate is connected at the mean diameters so:

$Dg_{bi} = OD_{bi} - t_{bi}$ $\qquad$ $Dg_{bu} = OD_{bu} - t_{bu}$

$R_{bi} = Dg_{bi} \cdot 0.5$ $\qquad$ $R_{bu} = Dg_{bu} \cdot 0.5$ $\qquad$ $c = R_{bi}$

$b = \left[R_{bu} - R_{bi}\right]$ $\qquad$ $d = R_{bu}$

$b = 43.942 \cdot mm$
$c = 157.163 \cdot mm$
$d = 201.105 \cdot mm$

$$\beta_{bi} = \left[\frac{3 \cdot \left[1 - \mu^2\right]}{R_{bi}^2 \cdot t_{bi}^2}\right]^{0.25} \qquad \beta_{bu} = \left[\frac{3 \cdot \left[1 - \mu^2\right]}{R_{bu}^2 \cdot t_{bu}^2}\right]^{0.25}$$

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$\beta_{bi} = 0.03321 \cdot \frac{1}{mm}$ $\beta_{bu} = 0.04425 \cdot \frac{1}{mm}$

$$D_{bi} = \frac{E_{bi} \cdot t_{bi}^{3}}{12 \cdot \left[1 - \mu^{2}\right]} \qquad D_{bu} = \frac{E_{bu} \cdot t_{bu}^{3}}{12 \cdot \left[1 - \mu^{2}\right]}$$

$D_{bi} = 14807087.3907 \cdot N \cdot mm$ $D_{bu} = 1248800.3898 \cdot N \cdot mm$

$$\lambda = 1 + \frac{\beta_{bu}^{3} \cdot D_{bu}}{4 \cdot \beta_{bi}^{3} \cdot D_{bi}} \qquad y = \left[E_{plate} \cdot t_{plate}^{3}\right] \cdot \ln\left[\frac{R_{bu}}{R_{bi}}\right]$$

$\lambda = 1.05$ $y = 78765731.99607 \cdot N \cdot mm$

$$\rho = \left[R_{bu} \cdot \frac{\beta_{bu} \cdot D_{bu}}{\left[2 \cdot \beta_{bi} \cdot D_{bi}\right] \cdot \left[2 - \frac{1}{\lambda}\right]} + R_{bi} + \frac{y}{48 \cdot \beta_{bi} \cdot D_{bi}}\right] \qquad \rho = 171.289 \cdot mm$$

$$\sigma_{tgbi} = \frac{\left[p_{bi} - p_{bu}\right] \cdot Dg_{bi}}{2 \cdot t_{bi}} \qquad \sigma_{tgbu} = \frac{p_{bu} \cdot Dg_{bu}}{2 \cdot t_{bu}}$$

$\sigma_{tgbi} = 1.485 \cdot \frac{N}{mm^{2}}$ $\sigma_{tgbu} = 50.384 \cdot \frac{N}{mm^{2}}$

$$\mu_{bi} = -\mu \cdot l_{bi} \cdot \frac{\sigma_{tgbi}}{E_{bi}} + \alpha_{bi} \cdot l_{bi} \qquad \mu_{bu} = -\mu \cdot l_{bu} \cdot \frac{\sigma_{tgbu}}{E_{bu}} + \alpha_{bu} \cdot l_{bu}$$

$\mu_{bi} = 5.674 \cdot mm$ $\mu_{bu} = 6.829 \cdot mm$

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$$N_{pbi} = \frac{\pi}{4} \cdot Dg_{bi}^{2} \cdot p_{bi}$$

$N_{pbi} = 88461.15005 \cdot N$

$N_{pbu} = 0 \cdot N$

$$F_{bi} = \frac{\dfrac{N_{pbi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} - \dfrac{\left(N_{pbu} + P_{bu}\right) \cdot l_{bu}}{E_{bu} \cdot A_{bu}} - \mu_{bu} - \dfrac{P_{bu} \cdot b^{2}}{16 \cdot \pi \cdot \rho \cdot \beta_{bi} \cdot D_{bi}}}{\dfrac{-l_{bi}}{E_{bi} \cdot A_{bi}} - \dfrac{2 \cdot b^{2}}{16 \cdot \pi \cdot \rho \cdot \beta_{bi} \cdot D_{bi}} - \dfrac{l_{bu}}{E_{bu} \cdot A_{bu}}}$$

$F_{bi} = 231189.97836 \cdot N$

$N_{bi} = N_{pbi} + F_{bi}$

$N_{bi} = 319651.12841 \cdot N$

$N_{bu} = N_{pbu} + P_{bu} - F_{bi}$

$N_{bu} = -184271.92568 \cdot N$

$$\sigma_{axbi} = \frac{N_{bi}}{A_{bi}}$$

$$\sigma_{axbu} = \frac{N_{bu}}{A_{bu}}$$

$\sigma_{axbi} = 35.047 \cdot MPa$

$\sigma_{axbu} = -35.163 \cdot MPa$

$F_{bu} = N_{bu} - N_{pbu}$

$F_{bu} = -184271.92568 \cdot N$

$$M_{obi} = \frac{\left(P_{bu} - 2 \cdot F_{bi}\right) \cdot b}{4 \cdot \pi \cdot \rho}$$

$M_{obi} = -8481.48294 \cdot N$

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$$M_{obu} = \frac{M_{obi} \cdot \left[\beta_{bu} \cdot D_{bu}\right]}{\left[2 \cdot \beta_{bi} \cdot D_{bi}\right] \cdot \left[2 - \frac{1}{\lambda}\right]} \qquad M_{obu} = -455.02307 \cdot N$$

$$\theta = 12 \cdot \frac{\frac{\left[P_{bu} - 2 \cdot F_{bi}\right] \cdot b}{4 \cdot \pi} - M_{obu} \cdot R_{bu} - M_{obi} \cdot R_{bi}}{y} \qquad \theta = -0.247 \cdot deg$$

$$\theta = \frac{M_{obi}}{4 \cdot \beta_{bi} \cdot D_{bi}} \qquad \text{(XXIII)} \qquad \theta = -0.247 \cdot deg$$

$$M = \left[\frac{\left[P_{bu} - 2 \cdot F_{bi}\right] \cdot b}{4 \cdot \pi} - M_{obu} \cdot R_{bu} - M_{obi} \cdot R_{bi}\right]$$

Moment in the plate

$$M = -28306.13499 \cdot N \cdot mm$$

$$Q_{obi} = \frac{\beta_{bu} \cdot M_{obu}}{\lambda} \qquad \text{(XVIII)} \qquad Q_{obu} = -Q_{obi} \qquad \text{(XIII)}$$

$$Q_{obi} = -19.179 \cdot \frac{N}{mm} \qquad Q_{obu} = 19.179 \cdot \frac{N}{mm}$$

$$Q_{olbi} = \frac{-\beta_{bi} \cdot M_{obi} + Q_{obi}}{2} \qquad Q_{olbi} = 131.229 \cdot \frac{N}{mm} \qquad \text{(V)}$$

$$Q_{orbi} = Q_{obi} - Q_{olbi} \qquad Q_{orbi} = -150.408 \cdot \frac{N}{mm} \qquad \text{(IIa)}$$

$$M_{olbi} = \frac{2 \cdot \beta_{bi} \cdot M_{obi} - Q_{obi}}{4 \cdot \beta_{bi}} \qquad M_{olbi} = -4096.34413 \cdot N \qquad \text{(VI)}$$

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$$M_{orbi} = M_{olbi} - M_{obi} \qquad M_{orbi} = 4385.1388 \cdot N \qquad \text{(Ia)}$$

$$\sigma_{tan} = \frac{6 \cdot M}{t_{plate}^{2} \cdot c \cdot \ln\left[\frac{d}{c}\right]} \qquad \sigma_{radbu} = \frac{-6 \cdot M_{obu}}{t_{plate}^{2}} \qquad \sigma_{lbi} = \frac{6 \cdot M_{olbi}}{t_{bi}^{2}}$$

$$\sigma_{radbi} = \frac{6 \cdot M_{obi}}{t_{plate}^{2}} \qquad \sigma_{bu} = \frac{6 \cdot M_{obu}}{t_{bu}^{2}} \qquad \sigma_{rbi} = \frac{6 \cdot M_{orbi}}{t_{bi}^{2}}$$

$$\tau_{bu} = \frac{F_{bu}}{2 \cdot \pi \cdot R_{bu} \cdot t_{plate}}$$

$$\tau_{bi} = \frac{-F_{bi}}{2 \cdot \pi \cdot R_{bi} \cdot t_{plate}}$$

DETERMINED STRESSES

$\sigma_{tan} = -30.43864 \cdot MPa$ Plate

$\sigma_{radbi} = -353.395 \cdot MPa$ Plate

$\sigma_{radbu} = 18.959 \cdot MPa$ Plate

$\tau_{bi} = -19.51 \cdot MPa$ Plate

$\tau_{bu} = -12.153 \cdot MPa$ Plate

$\sigma_{lbi} = -270.90543 \cdot MPa$ Core left

$\sigma_{rbi} = 290.00443 \cdot MPa$ Core right

$\sigma_{tgbi} = 1.485 \cdot MPa$ Core

$\sigma_{axbi} = 35.047 \cdot MPa$ Core

$\sigma_{bu} = -155.43519 \cdot MPa$ Jacket

$\sigma_{tgbu} = 50.384 \cdot MPa$ Jacket

$\sigma_{axbu} = -35.163 \cdot MPa$ Jacket

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EQUIVALENT STRESS

$$\sigma_{plbi} = \sqrt{\sigma_{tan}^2 + \sigma_{radbi}^2 - \sigma_{tan} \cdot \sigma_{radbi} + 3 \cdot \tau_{bi}^2} \qquad \sigma_{plbi} = 340.881 \cdot MPa$$

$$\sigma_{plbu} = \sqrt{\sigma_{tan}^2 + \sigma_{radbu}^2 - \sigma_{tan} \cdot \sigma_{radbu} + 3 \cdot \tau_{bu}^2} \qquad \sigma_{plbu} = 48.022 \cdot MPa$$

$S1_{bi} = -0.5 \cdot p_{bi}$ $\qquad$ $S1_{bu} = -0.5 \cdot p_{bu}$

$S2_{bi} = \sigma_{tgbi}$ $\qquad$ $S2_{bu} = \sigma_{tgbu}$

$S3_{rbi} = \sigma_{axbi} + \sigma_{rbi}$ $\qquad$ $S3_{bu} = \sigma_{axbu} + \sigma_{bu}$

$S3_{lbi} = \sigma_{axbi} + \sigma_{lbi}$

$$\sigma_{eqrbi} = \sqrt{0.5 \cdot \left[\left[S1_{bi} - S2_{bi}\right]^2 + \left[S2_{bi} - S3_{rbi}\right]^2 + \left[S3_{rbi} - S1_{bi}\right]^2\right]}$$

$$\sigma_{eqlbi} = \sqrt{0.5 \cdot \left[\left[S1_{bi} - S2_{bi}\right]^2 + \left[S2_{bi} - S3_{lbi}\right]^2 + \left[S3_{lbi} - S1_{bi}\right]^2\right]}$$

$$\sigma_{eqrbi} = \sqrt{0.5 \cdot \left[\left[S1_{bi} - S2_{bi}\right]^2 + \left[S2_{bi} - S3_{rbi}\right]^2 + \left[S3_{rbi} - S1_{bi}\right]^2\right]}$$

$$\sigma_{eqbu} = \sqrt{0.5 \cdot \left[\left[S1_{bu} - S2_{bu}\right]^2 + \left[S2_{bu} - S3_{bu}\right]^2 + \left[S3_{bu} - S1_{bu}\right]^2\right]}$$

$\sigma_{eqrbi} = 324.598 \cdot MPa$ $\qquad$ $\sigma_{eqlbi} = 236.323 \cdot MPa$

$\sigma_{eqbu} = 219.991 \cdot MPa$

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$$w_{bi} = \frac{-Q_{obi}}{8 \cdot \beta_{bi}^{3} \cdot D_{bi}} \qquad w_{bi} = 0.00442 \cdot mm$$

$$w_{bu} = \frac{-\left[\beta_{bu} \cdot M_{obu} + Q_{obu}\right]}{2 \cdot \beta_{bu}^{3} \cdot D_{bi}} \qquad w_{bu} = 0.00088 \cdot mm$$

$$\delta_{ax} = b \cdot \sin(\theta) \qquad \delta_{ax} = -0.1895 \cdot mm$$

$$\delta l_{bi} = \frac{N_{bi} \cdot l_{bi}}{E_{bi} \cdot A_{bi}} + \mu_{bi} \qquad \delta l_{bi} = 6.153 \cdot mm$$

$$\delta l_{bu} = \frac{N_{bu} \cdot l_{bu}}{E_{bu} \cdot A_{bu}} + \mu_{bu} \qquad \delta l_{bu} = 6.343 \cdot mm$$

$$d_{ax} = \delta l_{bi} - \delta l_{bu} \qquad d_{ax} = -0.1895 \cdot mm \qquad \text{(XXIX)}$$

For use in pipestress programme:

$$Spring_{rate} = \left|\frac{F_{bi}}{\delta_{ax}}\right| \qquad Spring_{rate} = 1220017.39961 \cdot \frac{N}{mm}$$

Stoomwezen B.V. CBA